

$$1. \int 3x^2 \sqrt{x^3-7} \, dx$$

$$u = x^3 - 7 \quad \int 3x^2 \cdot u^{1/2} \cdot \frac{du}{3x^2}$$

$$\frac{du}{dx} = 3x^2 \quad \int u^{1/2} du$$

$$dx = \frac{du}{3x^2} \quad \frac{2}{3} u^{3/2} + C$$

$$\frac{2}{3} (x^3 - 7)^{3/2} + C$$

$$5. \int \frac{3x^2}{\sqrt[3]{2x^3+11}} \, dx$$

$$u = 2x^3 + 11 \quad \int \frac{3x^2}{u^{1/3}} \cdot \frac{du}{6x^2}$$

$$\frac{du}{dx} = 6x^2 \quad \frac{1}{2} \int u^{-1/3} du$$

$$dx = \frac{du}{6x^2} \quad \frac{3}{4} u^{2/3} + C$$

$$\frac{3}{4} (2x^3 + 11)^{2/3} + C$$

$$2. \int 3 \csc x \, dx$$

$$-3 \ln |\csc x + \cot x| + C$$

$$3. \int \frac{1 - \sin^2(2x)}{\sin(2x) \cos(2x)} \, dx$$

$$\int \frac{\cos^2(2x)}{\sin(2x) \cos(2x)} \, dx$$

$$\int \cot(2x) \, dx$$

$$u = 2x \quad \int \cot u \cdot \frac{du}{2}$$

$$\frac{du}{dx} = 2 \quad \frac{1}{2} \int \cot u \, du$$

$$dx = \frac{du}{2} \quad \frac{1}{2} \ln |\sin u| + C$$

$$\frac{1}{2} \ln |\sin 2x| + C$$

$$4. \int 4x \tan(5x^2) \, dx$$

$$u = 5x^2 \quad \int 4x \tan u \cdot \frac{du}{10x}$$

$$\frac{du}{dx} = 10x \quad \frac{2}{5} \int \tan u \, du$$

$$dx = \frac{du}{10x} \quad -\frac{2}{5} \ln |\cos u| + C$$

$$-\frac{2}{5} \ln |\cos 5x^2| + C$$

$$6. \int \csc^2 x \cot^2 x \, dx$$

$$u = \cot x \quad \int \csc^2 x \cdot u^2 \cdot \frac{du}{-\csc^2 x}$$

$$\frac{du}{dx} = -\csc^2 x \quad -\int u^2 \, du$$

$$dx = \frac{du}{-\csc^2 x} \quad -\frac{1}{3} u^3 + C$$

$$-\frac{1}{3} \cot^3 x + C$$

$$7. \int (21x - 28)(3x^2 - 8x)^4 \, dx$$

$$u = 3x^2 - 8x \quad \int 21x - 28 \cdot u^4 \cdot \frac{du}{6x - 8}$$

$$\frac{du}{dx} = 6x - 8 \quad \int 7(3x - 4) u^4 \cdot \frac{du}{2(3x - 4)}$$

$$dx = \frac{du}{6x - 8} \quad \frac{7}{2} \int u^4 \, du$$

$$\frac{7}{10} u^5 + C$$

$$\frac{7}{10} (3x^2 - 8x)^5 + C$$

$$8. \int \frac{4 \sin(3x)}{\cos^2(3x)} \, dx$$

$$4 \int \sec(3x) \tan(3x) \, dx$$

$$u = 3x \quad 4 \int \sec u \tan u \cdot \frac{du}{3}$$

$$\frac{du}{dx} = 3 \quad \frac{4}{3} \sec u + C$$

$$dx = \frac{du}{3} \quad \frac{4}{3} \sec(3x) + C$$

$$9. \int 5x^2 \sqrt{2x^3-1} \, dx$$

$$u = 2x^3 - 1 \quad \int 5x^2 \cdot u^{1/2} \cdot \frac{du}{6x^2}$$

$$\frac{du}{dx} = 6x^2 \quad \frac{5}{6} \int u^{1/2} du$$

$$dx = \frac{du}{6x^2} \quad \frac{10}{18} u^{3/2} + C$$

$$\frac{5}{9} (2x^3-1)^{3/2} + C$$

$$10. \int 2x \sin^3(3x^2) \cos(3x^2) \, dx$$

$$u = \sin(3x^2) \quad \int 2x \cdot u^3 \cdot \cos(3x^2) \cdot \frac{du}{6x \cos(3x^2)}$$

$$\frac{du}{dx} = \cos(3x^2) \cdot 6x \quad \frac{1}{3} \int u^3 du$$

$$dx = \frac{du}{6x \cos(3x^2)} \quad \frac{1}{12} u^4 + C$$

$$\frac{1}{12} \sin^4(3x^2) + C$$

$$11. \int \frac{\sin(2x) \tan^2(2x) + \sin(2x)}{\frac{1}{\csc(2x)}} \, dx$$

$$\int \frac{\cancel{\sin(2x)} (\tan^2(2x) + 1)}{\cancel{\sin(2x)}} \, dx$$

$$\int \sec^2(2x) \, dx$$

$$u = 2x \quad \int \sec^2 u \cdot \frac{du}{2}$$

$$\frac{du}{dx} = 2 \quad \frac{1}{2} \int \sec^2 u \, du$$

$$dx = \frac{du}{2} \quad \frac{1}{2} \tan u + C$$

$$\frac{1}{2} \tan(2x) + C$$

$$12. \int \frac{2x \sec^3(4x^2) \sin(4x^2)}{5 \cos(4x^2)} \, dx$$

$$\frac{2}{5} \int x \sec^3(4x^2) \tan(4x^2) \, dx$$

$$u = \sec(4x^2)$$

$$\frac{du}{dx} = \sec(4x^2) \tan(4x^2) \cdot 8x$$

$$dx = \frac{du}{8x \sec(4x^2) \tan(4x^2)}$$

$$\frac{2}{5} \int \cancel{x} \cdot u^2 \cdot \sec(4x^2) \tan(4x^2) \cdot \frac{du}{8x \sec(4x^2) \tan(4x^2)}$$

$$\frac{1}{20} \int u^2 \, du$$

$$\frac{1}{60} u^3 + C$$

$$\frac{1}{60} \sec^3(4x^2) + C$$

$$13. \int 3x e^{4x^2-5} dx$$

$$u = 4x^2 - 5 \quad \int 3x \cdot e^u \cdot \frac{du}{8x}$$

$$\frac{du}{dx} = 8x \quad \frac{3}{8} \int e^u du$$

$$dx = \frac{du}{8x} \quad \frac{3}{8} e^u + C$$

$$\frac{3}{8} e^{4x^2-5} + C$$

$$14. \int \frac{e^{\tan(2x)}}{\cos^2(2x)} dx$$

$$\int e^{\tan(2x)} \sec^2(2x) dx$$

$$u = \tan(2x) \quad \int e^u \cdot \sec^2(2x) \cdot \frac{du}{2\sec^2(2x)}$$

$$\frac{du}{dx} = \sec^2(2x) \cdot 2 \quad \frac{1}{2} \int e^u du$$

$$dx = \frac{du}{2\sec^2(2x)} \quad \frac{1}{2} e^u + C$$

$$\frac{1}{2} e^{\tan(2x)} + C$$

$$15. \int 4x^2 \cdot 3^{5x^3+2} dx$$

$$u = 5x^3 + 2 \quad \int 4x^2 \cdot 3^u \cdot \frac{du}{15x^2}$$

$$\frac{du}{dx} = 15x^2 \quad \frac{4}{15} \int 3^u du$$

$$dx = \frac{du}{15x^2} \quad \frac{4}{15} \cdot \frac{3^u}{\ln 3} + C$$

$$\frac{4}{15} \cdot \frac{3^{5x^3+2}}{\ln 3} + C$$

$$16. \int \frac{x^3 e^{3x^4} \cdot 5^{e^{3x^4}}}{6} dx$$

$$u = e^{3x^4} \quad \frac{1}{6} \int x^3 e^{3x^4} \cdot 5^u \cdot \frac{du}{12x^3 e^{3x^4}}$$

$$\frac{du}{dx} = e^{3x^4} \cdot 12x^3 \quad \frac{1}{72} \int 5^u du$$

$$dx = \frac{du}{12x^3 \cdot e^{3x^4}} \quad \frac{1}{72} \cdot \frac{5^u}{\ln 5} + C$$

$$\frac{1}{72} \cdot \frac{5^{e^{3x^4}}}{\ln 5} + C$$